WEAK SEPARABILITY, EFFICIENCY AND SECOND-BEST: A GENERAL FORMULATION

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Abstract

Kaplow (1996) shows that, when utility is weakly separable and a nonlinear income tax is available, no correction of the first best rule for the optimal provision of public goods is required. We provide a general formulation for this results, and use it to deal with some other known cases in which the first best rule carries over to the second best.

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JEL classification: H21, H23

1. Introduction

According to the general theorem of second best and to the second fundamental theorem of welfare economics, outside of the first best world in which only lump sum taxes are used for redistributive purposes, an increase in social welfare can require the introduction of additional distortions. In a second best world, the common efficiency rules developed for the first best case do not necessarily provide a well-grounded guidance to policy.

However, there are some circumstances in which first best rules carry over to the second best. A well known example is the result that, in the Mirrlees framework where individuals differ only in their ability to earn income, a distortion of relative prices through differentiated commodity taxes is not called for when an optimal nonlinear tax is in place and utility is separable in commodities and leisure/labor (Atkinson and Stiglitz, 1976).

Similarly, Kaplow (1996) proves that, under the conditions of identical and weakly separable preferences, if a nonlinear change in the income tax is possible, then the optimal provision of a public good is determined by the first best efficiency rule[1]. What is remarkable in Kaplow's contribution is that it provides a simple way to deal with this and related problems. Moreover, the result does not require that the income tax is optimal in any sense.

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[1]The claim that under these conditions public programs should not depend on distributional objectives, and only efficiency should count, can be traced back to Hylland and Zeckhauser (1979)

The idea is that, whatever the initial distribution and the income tax, any effect the provision of a public good can have on individuals can be "neutralized" by a suitable change of the income tax schedule. At each income level, the income tax is increased by an amount equal to the benefit that the individual at that income level receives from the public good. In this way, utility is unchanged for all income levels, hence there is no change in labor supply. The change in total tax revenue corresponds to the sum of the benefits individuals obtain from the public good; if this exceeds costs—i.e. if Samuelson's condition applies—then it is possible to make public good provision Pareto improving.

In this paper we provide a general formulation of Kaplow's result (section 2), and use it to restate some well known conclusions in the optimal taxation literature (section 3). No new result is presented: what is new is the strategy of proof, which is remarkably simpler than what is usually found in public economics textbooks. In the concluding section we consider the extension of the result to the case of "mild" heterogeneity among agents.

2. General formulation

All individuals have identical preferences, and differ only in their "productivity" w, i.e. an unobservable vector of characteristics which affects their ability to transform "effort" ℓ into income. Gross income $y = y(w,\ell)$ is perfectly observable, while w and ℓ cannot be observed. With no loss of generality, we make the common assumption that $y = w\ell$. The government can levy a nonlinear income tax, and $z = z(y)$ represents net-of-tax income.

Each individual selects a variable $x \in B$ which affects her utility ; this can be thought of as a bundle of consumption goods, in which case $B \subseteq \Re^n_+$; the set B is a function of net income z, but it is not affected by w or ℓ directly. We consider a policy instrument denoted by k, affecting utility and/or the choice set B. To summarize, the individual chooses $x \in B(k,z)$ and her utility can be written as

$$u(\phi(x,k),\ell). \tag{1}$$

Let $v(z,k) = \max_{x\in B(k,z)} \phi(x,k)$; the optimal choice of ℓ solves

$$\max_{\ell}\ u(v(z(w\ell),k),\ell). \tag{2}$$

Let $\tilde{z}(z,k,k_0)$, usually referred to as the *(indirect) compensation function*, be defined as

$$v(\tilde{z}(z,k,k_0),k) \equiv v(z,k_0); \tag{3}$$

we have $\tilde{z}(z,k,k) = z$ for all k. The value of $\tilde{z}$ can be calculated by the government because the effect of the change in k for each individual depends only on her net income; this would not be the case if B or ϕ (and therefore v) depended directly on w or ℓ.

If the government accompanies the "reform" $k_0 \to k$ with a tax change from $z(.)$ to $\tilde{z}(z(.),k,k_0)$ then, independently of the selected k, the maximization problem for type-w individual is

$$\max_{\ell} u(v(z(w\ell),k_0),\ell) \tag{4}$$

hence for all w the optimal choice of ℓ is unaffected by the reform. Let $Z(w)$ be the level of pre-reform net income selected by type-w individual; the income tax of this individual is reduced by $\tilde{z}(Z(w),k,k_0) - Z(w)$, which is equal to her compensating income variation for the reform.

The reform $k_0 \to k$ plus the tax change leave all individuals at the initial utility level, while bringing about a change in tax revenue. The additional revenue accruing to the government net of the cost $c(k) - c(k_0)$ of implementing the reform is

$$\int_W [Z(w) - \tilde{z}(Z(w), k, k_0)] f(w) dw - [c(k) - c(k_0)]. \quad (5)$$

Therefore, if the sum of compensating variations exceeds the cost of the reform, the government gets a surplus, which can be used to obtain a Pareto improvement. We can state the following

Proposition 1. *Given any income tax $z(.)$, a policy represented by k_0 is optimal only if* (5) *is nonpositive for all k, i.e. only if k_0 solves*

$$\min_k \int_W \tilde{z}(Z(w), k, k_0) f(w) dw + c(k). \quad (6)$$

□

To summarize: if individuals have identical preferences and differ only in their ability to earn income, utilities are (weakly) separable in leisure/effort, and a nonlinear change of the income tax is feasible, there is no difference between the first and the second best criterion.

3. Specialization of the result

We now show how the framework developed above can be specialized to deal with some well known results in the literature on optimal taxation in second best. This should make clear that these are all instances of the same result.

3.1. Public goods

We specialize the problem by letting k indicate the quantity of a public good. We have $B(k,z) = \{x | px \leq z\}$ and $c(k) = p_G k$ where p_G is the (linear) production cost of the public good. By assuming that the public good affects (only) ϕ, we allow the benefit to vary with income and can be complementary/substitutable w.r.t. private goods, though we rule out complementarity/substitutability with leisure.

The first order condition for the maximization of (6) is

$$\int_W \frac{v_k}{v_z} f(w) dw = p_G \quad (7)$$

which is Samuelson's condition. This is the result proved by Christiansen (1981) and restated by Kaplow (1996).

3.2. Externalities

Suppose that individual utility depends on an externality E, so that $\phi(x, E)$. As in the case of the public good, we are considering the case that the effect of the externality does not depend directly on individual ability or on labor supply, though it can depend, and will depend in general, on consumption of other goods, hence on the income level of the individual. Our setting can include e.g. the case in which the externality is created by rich people and affects mainly poor people, or other cases in which it has a distributive dimension.

We assume (without loss of generality) that the externality is a function of X_1, the aggregate consumption of commodity 1. The government can affect this consumption choice: in the case in which regulation takes the form of a Pigouvian tax k[2]

$$B(k,z) = \{x|kx_1 + px \leq z\} \tag{8}$$

so that an individual with net income z chooses $x = x(z,k)$; the level of externality as a function of k is

$$X_1(k) = \int_W x_1(Z(w),k) f(w) dw. \tag{9}$$

We have $v(z,k) = \phi(x(z,k), X_1(k))$, and use Proposition 1 to find the optimal level of k. Considering that $d\tilde{z}/dk = -x_1$ and that $c(k) = -kX_1$, we have the following first order condition

$$\int_W \left[x_1(Z(w),k) + \frac{\phi_E}{v_z} \frac{dX_1}{dk} \right] f(w) dw - X_1 - k\frac{dX_1}{dk} = 0 \tag{10}$$

or, simplifying,

$$\int_W \frac{\phi_E}{v_z} f(w) dw = k \tag{11}$$

so that the tax should be set equal to the (unweighted) sum of the the marginal effects of the externality. Neither concern for distribution nor the fact that income taxation is distortionary justify the dismissal of the first best condition.

This is essentially the same conclusion of Cremer, Gahvari and Ladoux (1998) that the optimal tax on the externality generating good is strictly Pigouvian when individuals have identical marginal rates of substitution at any given consumption bundle[3].

3.3. *Commodity taxation*

In this case, $B(k,z) = \{x|(p+k)x \leq z\}$ with $k \in \Re^n_+$, and $\phi(x)$. We set $k_0 = 0$, and consider that

$$Z(w) = e(u_0(w), p) = ph(u_0(w), p) \tag{12}$$

$$\tilde{z}(Z(w),k,0) = e(u_0(w), p+k) = (p+k)h(u_0(w), p+k) \tag{13}$$

where $u_0(w) = \max_\ell u(v(z(w\ell),0),\ell)$ is the initial level of utility, e is the expenditure function and h is the vector of compensated demands. We have $c(0) = 0$ and

$$c(k) = -k \int_W h(u_0(w), p+k) f(w) dw \tag{14}$$

(the "cost" is in this case the revenue from commodity taxes). Thus, expression (5) can be written as

$$\int_W [e(u_0(w), p) - ph(u_0(w), p+k)] f(w) dw. \tag{15}$$

[2]Similarly, we could examine the case of a quantity constraint by considering $B(k,z) = \{x|px \leq z \text{ and } x_1 \leq k\}$.

[3]The possibility to apply the result to the case of externalities was mentioned by Kaplow (1996). With minor modifications, our conclusion corresponds to the claim of Kaplow and Shavell (1994) on the irrelevance of distributional consideration when externalities are corrected through liability rules.

By the definition of expenditure function, we have for each w

$$e(u_0(w), p) \leq ph(u_0(w), p+k) \tag{16}$$

so that expression (15) is always nonpositive. Indeed, because of homogeneity of compensated demand functions, (16) becomes an equality—therefore (15) is zero—when k is proportional to p. In conclusion, under the stated conditions it is never optimal to have differentiated commodity taxation: we cannot increase welfare by distorting prices. This is the result in Atkinson and Stiglitz (1976), extended to the case in which the income tax is not necessarily optimal.

4. Extension to the case of heterogeneous individuals

The analysis above, like most of optimal taxation literature, assumes that individual have identical preferences and differ only in their ability to earn income.

It is not possible a straightforward extension of Proposition 1 to the case of heterogeneous individuals, because there is no way to find a change in the income tax that compensates individuals with the same income levels if the reform affects them differently.

However, the results presented here can be easily extended to the case in which heterogeneity is limited to the choice between leisure and consumption, while preferences on the allocation of income among different uses are still identical across individuals.

This is easily shown considering that in this case utility is in the form $u^h(v(z,k),\ell)$ where h identifies the preference "type": equation (3) defining $\tilde{z}$ will not depend on h, hence the argument and the formalization are exactly the same as in the case of homogeneous preferences[4].

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[4]Blomquist and Christiansen (2003) advance a taxonomy of cases of preference heterogeneity, and prove that, when individuals differ only in their preference for leisure, Atkinson and Stiglitz's result on the redundancy of differentiated commodity taxation still applies.