From household to individual's welfare: does the Lorenz criteria still hold? Theory and Evidence from French Data*

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Abstract

Consider an income distribution among households of the same size in which individuals, equally needy from the point of view of an ethical observer, are treated unfairly. Individuals are split into two types, the dominant and the dominated. We look for conditions under which the Generalized Lorenz test is preserved from household to individual level. We find that the concavity of the share of expenditures devoted to public goods relatively to household income and the concavity of the share of expenditure devoted to private goods of the dominated individual are sufficient conditions. This result is extended to the case of heterogeneous populations (singles and couples), when more complex Lorenz comparisons are involved. In the second part of the paper, we propose a new method to identify the intra-family sharing rule. The double concavity assumption is then non-parametricaly tested . Using data on French households, the double concavity is not rejected, this goes in favor of the preservation property of the Lorenz criteria.

1 Introduction

Recent literature emphasizes the importance of omitting intra-household inequality in normative analysis. Haddad and Kanbur (1990) find that when an additive inequality index is used to measure the level of inequality inside a population, then a serious downward bias appears because intra-household inequality is omitted. Taking into account intra-household inequality in normative analysis would be straightforward if individual's welfare was directly observed. Unfortunately this is not the case as incomes and consumptions are generally collected at the household level, moreover economies of scales need to be controlled.

In this paper, we question whether or not Lorenz-type comparisons are biased when ignoring the effect of intra-household inequality. If households (homogeneous in their composition) shared equally their resources, then it would be sufficient to resort to Lorenz dominance at the household level in order to compare inequality and welfare at the individual level. Peluso and Trannoy (2004) show that we may enlarge the validity of some very well known criteria of dominance of the Lorenz type beyond the strict case of pure equality between members of the household. Their starting point is that although individuals have the same needs from the point of view of an ethical observer, each household contains *dominant* individuals, advantaged in their private consumption with respect to *dominated* individuals. Under this assumption, the Lorenz comparisons between households are meaningful for the evolution of inequality among individuals if and only if the part of private expenditure devoted to the dominated individuals remains a constant share of the household income. This part must represent a concave function of household income whether we are interested in the comparisons brought by the Generalized Lorenz test (Shorrocks (1983)), which mixes both the size and the distribution dimensions in the appraisal of welfare.

These results, albeit interesting, do not fully allow us to test empirically the sensibility of the main assumptions. One important aspect has not been taken into account in the previous work: the presence of family public goods. It is well accepted that individuals living together generate public consumption of goods, altruism and externalities and the impact of these phenomena on the individual well-being cannot be dismissed. We assume here both intra-household public consumption of goods and a form of discrimination in each household. We show that an *additional* sufficient condition to get the preservation of the Generalized Lorenz test at the individual level is provided by the concavity of the part of expenditures devoted to public good relatively to household income. The richer the household are, the lesser the part devoted to public good must be. The critical property we identify is then the concavity of two sharing functions*:* a *public sharing function,* expressing the expenditure in public goods as a function of the household income, and a *private sharing function,* indicating the private expenditure of the disadvantaged individual as a function of the total sum devoted in private goods in the household. These assumptions mean that poorest households are the more egalitarian too. An interpretation could be that when a household becomes richer, the share of global income used for personal expenditures becomes more and more important, and the individual with the strongest bargaining power at level of household takes the highest advantage.

A second extension encompasses the diversity of the population. The mentioned results concern a population homogeneous regarding the size and the composition of the households. It is difficult to maintain such an assumption in any empirical investigation, where the differences of individuals choice across family status often provide sources of identification. Here, to make things simple, but w.l.o.g, we consider populations composed of couples and singles. The appropriate criteria for welfare comparisons are a test pointed out by Bourguignon (1989) and the Sequential Generalized Lorenz test, proposed by Atkinson and Bourguignon (1987). The first criterion is based on the assumption that the marginal utility of an euro received by a couple is higher than the marginal utility of an euro received by a single individual. The Atkinson and Bourguignon test also assumes that this difference of marginal utility becomes less and less relevant when income increases. These assumptions may be translated in terms transfer principles:respectively, the social welfare is increased when a single makes a transfer of income in favor of a couple with less income, and the social welfare increases all the more as progressive transfers are performed among couples rather than among singles, others things being equal.

We exhibit conditions on intra-household distribution which convert the Bourguignon (1989) dominance criterion among homogeneous households into the Generalized Lorenz dominance at the level of individuals. We also show that it is impossible, in general, to produce a similar result for the Sequential Generalized dominance test, that consequently results unappealing when intra-household discrimination is a relevant phenomenon.

These conditions may be served as testable restrictions in an econometric analysis. Using the French Household Expenditure Survey Data (FHES *Enquête Budget des Familles,* year 2000), we estimate non-parametrically the intra-family share of income devoted to public good as well as the share of the dominated individual. Our identification strategy is to assume that single individuals and members of couples of the same sex have the same taste for clothes. A double concavity test is then implemented by checking the sign of the second derivatives of the sharing rule with respect to household's income. One has to justify in testing the shape of the sharing functions that we do not control for other variables such as wage rates or other variables. The share of an household in private expenditure depends on many factors. Two main factors may be defended on some ethical grounds.

The first factors are related to needs. If you are taller than your partner, you have some claim for a higher share in food expenditure, for instance. Since, in western countries, food expenditures do no represent more than 20 % of the household budget, a difference of 20% in calorie need may vindicated a difference of 4% of the share in private expenditure, which admittedly belongs in the error margin. On the other hand, one may surely defend significative differences of food need in underdeveloped countries. We conclude that our assumption of identical claim to the resources from a need perspective is a sensible assumption in a developed country like France.

The second factors are linked to the notion of merit or talent. A good variable to capture differences with respect to this factor is the market wage rate. Different philosophical points of view have been defended regarding the pros and cons of claims on a bigger share of resources based on a higher talent. Here we suppose that the ethical observer does not support the view that the market wage rate has something to do with the intra-household distribution of resources. Based on that premise, we deduce that it is correct not to control in the estimation analysis of the sharing function for differences between the two partners in market wage rates.

In Section 2 we sketch the model and we prove the results about the link between the welfare comparisons among homogeneous households and among individuals. In the section 3 we study the case with heterogeneous households. In Section 4 we present the empirical study. We discuss further extensions and possible developments in Section 5, that concludes the paper. Tables and proofs are collected in the Appendix.

2 The model and welfare analysis

2.1 Lorenz tests for a homogeneous population

We focus on a population composed of n couples (indexed by $i = 1, ..., n$, with $n \geq 2$) different only for their income. Let $\mathbf{Y}^c$ designate a generic income vector for couples, rearranged in an increasing way. Its feasible set $\mathbb{Y}_n$ is

$$\mathbb{Y}_n = \left\{ \mathbf{Y}^c \in \mathbb{R}^n_+ \mid Y^c_1 \leq Y^c_2 ... \leq Y^c_n \right\}.$$

The welfare quasi-order in which we are interested is the Generalized Lorenz criterion (GL) (see Shorrocks (1983)). For the sake of completeness, we recall this criterion.

Definition 1 ***GL dominance.*** *Given* $\mathbf{Y}^c, \mathbf{Y}^{c\prime} \in \mathbb{Y}_n$,

$\mathbf{Y}^c$ dominates $\mathbf{Y}^{c\prime}$ according to the Generalized Lorenz test, denoted by $\mathbf{Y}^c \succcurlyeq_{GL} \mathbf{Y}^{c\prime}$, if:

$$\frac{1}{n}\sum_{i=1}^{k} Y^c_i \geq \frac{1}{n}\sum_{i=1}^{k} Y^{c\prime}_i \quad for\ k = 1, .., n.$$

The idea that all *individuals* have the same needs is translated, "dominance approach" to inequality, by evaluating the well-being guaranteed by an income x to any individual through the same 'utility' function $u(x)$. The GL test may be used in this case for welfare comparisons at the individual level: an *individual* income distribution $\mathbf{y}$ dominates $\mathbf{y}'$ according to the GL test if and only if

$$\sum_{j=1}^{2n} u(y_j) \geq \sum_{j=1}^{2n} u(y_j') \qquad (1)$$

for all the class of non-decreasing and concave utility function u. This standard result is completed by a *principle of transfers*: $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$ if and only if $\mathbf{y}$ can be obtained from $\mathbf{y}'$ by a finite sequence of *progressive transfers* (also named *Pigou-Dalton transfers*) or *increments*.[1]

2.2 Household consumption and individual incomes

The household model we adopt may be seen as the representation of the intra-household behavior of a generic couple made by a 'ethical observer', which takes into account two main features: the cooperation among the members of the couple, and, at the same time, a form of discrimination. We first describe the cooperation between household members by assuming that some part of the household income is spent for pure public goods. This allows to control for altruistic attitudes and externalities within the family. Let Y_i be the income of the household i, we designate by $g : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ a twice continuously differentiable function, identical across households, that gives the part of the household budget devoted to public goods. We assume that any *public sharing function* g introduced in this paper respects the following properties: $g(0) = 0$, $g(Y_i) \leq Y_i$ and $g'(Y_i) \in [0, 1]$,$\forall Y_i \geq 0$ (we exclude a decrease of the stock of public goods with income or the possibility of increase it by debt).

The remaining part of household income, $Y_i - g(Y_i)$ (henceforth denoted by $\tilde{Y}_i$) is shared between a *dominant* and a *dominated* individual, who spend this sum for private consumption. The dominated individual receives at most a sum equal to the amount

[1] See Marshall and Olkin, (1979), C.6, p. 28 and A.9.a, p. 123)

allowed to the dominant one. The amount $p_i = f_p(\tilde{Y}_i)$ received by the dominated individual in household i is described by a *private sharing function,* designated by $f_p : \mathbb{R}_+ \to \mathbb{R}_+$.[2] The function $f_p : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, identical across households and such that: $f_p(0) = 0$ and $f_p(x) \leq \frac{1}{2}x$, $\forall x \in \mathbb{R}_+$. Consequently, the amount r_i of private expenditure of the dominant type is:

$$r_i = f_r(\tilde{Y}_i) = \tilde{Y}_i - f_p(\tilde{Y}_i).$$

In the normative analysis, income or consumption of households may equivalently be used for welfare comparisons, when saving and prices variations are omitted (see Deaton and Zaidi (2002)). When we extend the analysis to individuals, if we neglect public consumption in the household, a simple definition of *individual* income naturally emerges as the part of the household budget devoted to each household member to her (or his) private goods. In the presence of public goods, no obvious definition comes out without additional assumptions. In an abstract way, the *individual income* y_{ij} summarizes in the household i the contribution of expenditure for public and private goods to the well-being of individual j

$$\begin{aligned} u(y_{ip}) &= u[g(Y_i), f_p(\tilde{Y}_i)] \\ u(y_{ir}) &= u[g(Y_i), f_r(\tilde{Y}_i)]. \end{aligned}$$

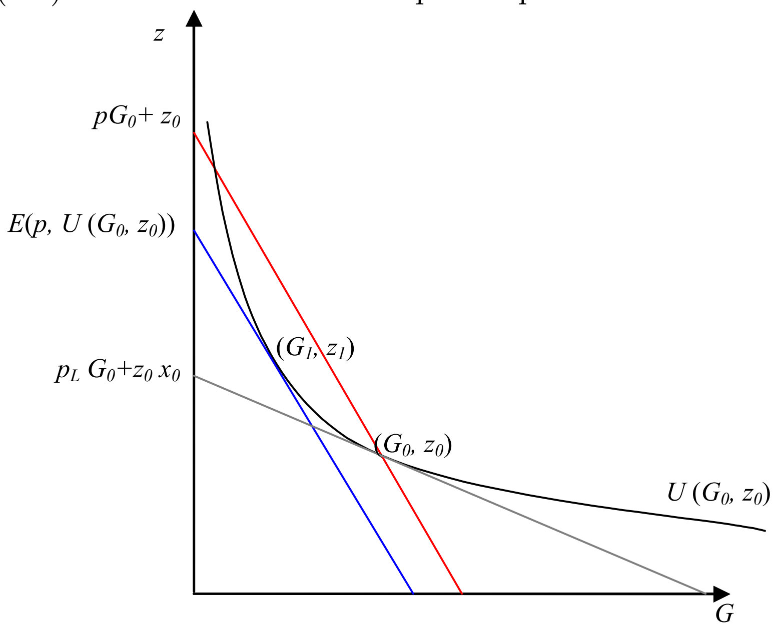
We define here the *individual income* y_{ij} of the household member $j = p$, r (that stay for poor and rich) living in the family i, as the income needed to a single in order to get the same consumption bundle than the individual of his (her) type living in a couple endowed with Y_i.

$$y_{ij} = \begin{cases} g(Y_i) + f_p\left(\tilde{Y}_i\right) & \text{if } j \text{ is a dominated type} \\ g(Y_i) + f_r(\tilde{Y}_i) & \text{if } j \text{ is a dominant type.} \end{cases} \tag{2}$$

[2]The sharing function could be seen as the reduced form of a structural model of the household, in which the private consumption is considered *conditionally* to a given expenditure in public goods (see for instance Blundell, Chiappori and Meghir (2002)).

Since this concept follows from the comparison of consumption opportunities, it does not require the invariance of individual preferences across the marital status. This is consistent with our empirical analysis, where we put very limited restrictions on individual preferences. In genereal, to compare the welfare of an individual living in couple (where some decisions of consumption are collective) with the welfare of an individual of the same type living alone is a non-trivial exercise. Different solutions may in fact emerge according to the context and the informational setup. In the following example, we show that at least three definition of *individual income* must be considered.

Example1 Let z be a Hicksian good (with unitary price) representing the private consumption of a given individual living in a couple. The public good is designated by G and let p ($\simeq 2$ in the figure below) be its market price. We suppose that the couple follows a classic scheme of contribution for the public good: if (G_0, z_0) represents the consumption bundle of a person in the household, the shape of his (her) indifference curve at this point equalizes the Lindhal price p_L.[3]



Three definitions of *individual income* in a couple

[3] We exclude here free riding as any other source of inefficiency.

The definition of *individual income* adopted in this paper is $x = pG_0 + z_0$, corresponding to the income needed to a single individual in order to have the bundle (G_0, z_0) in his (her) opportunity set. This concept just depends on household income and sharing rules. It does not require neither *preferences comparisons*, neither *situations comparisons*.[4] On one hand, this approach seems consistent with the heterogeneity of individual preferences across marital status allowed in this paper. On the other hand, it may induce an overestimation of the *individual incomes* of the household members. Assuming, for instance, that individual preferences are invariant w.r.t. marital status (focusing then on *situation comparisons*) we may define the *individual equivalent income* $E(p, U(G_0, z_0))$ as the income needed to a single in order to achieve the same utility level reached in (G_0, z_0). This expenditure is in general less than $pG_0 + z_0$, since a single with "regular" preferences chooses (G_1, z_1), cheaper than (G_0, z_0) by definition of Hicksian demand. The rationale of the overestimation of *individual incomes* adopted here w.r.t. *individual equivalent incomes* is rather intuitive: a lower income is sufficient to satisfy individuals when a restriction on their preferences is introduced. [5]

A third normative evaluation of the income of an individual living in a couple could be provided by introducing the Lindhal price p_L. In fact, $p_L G_0 + z_0$ represents the value of his (her) private goods and the value of the part of public good financed by him (her). This perspective is probably the more appropriate one when a sharing rule is determined in a context of divorce, assuming that the public good is dismissed at the market price and the resulting sum has to be shared among "ex". For this reason we call it *personal income*.

[4] That is the study of the choices of the same type of individual across different price-demographic situations, see Pollak (1991).

[5] *Observe that the two individual incomes defined above coincide if individual preferences have a null elasticity of substitution between "private" and "public" goods. Albeit trivial, this observation indicates an interesting way in order to extend the results of this paper, as we will clarify in the last section. (à faire)*

It emerges that a different context of application or a different informational setup may legitimate different definitions of individual income. We exclude the *personal income* since it totally neglect (by definition) the scale economies at the level of the household. We reject the *individual equivalent income* since it derives from a restriction on the informational setup not required in this paper, where we do not impose the full invariance of individual preferences across the marital status.

2.3 Welfare analysis: from couples to individuals

Our main results are related to the concavity of the sharing functions g and f_p. This property guarantees that welfare and inequality tests performed on households' incomes distributions are informative about the pattern of welfare and inequality at the individual level. More precisely, in our first proposition, we show that concavity of g and of f_p secures the preservation of the Generalized Lorenz ranking: an improvement in the distribution of household incomes in the sense of the generalized Lorenz test generates a similar improvement in the distribution of individual incomes.

Proposition 1 *If g and f_p are increasing and concave, then, for all $\mathbf{Y}, \mathbf{Y}' \in \mathbb{Y}_n$*

$$\mathbf{Y} \succcurlyeq_{GL} \mathbf{Y}' \Rightarrow \mathbf{y} \succcurlyeq_{GL} \mathbf{y}'.$$

Proof. See Appendix ■

3 Extension to a population of singles and couples

3.1 Lorenz tests for heterogeneous households

The welfare analysis developed in the previous section is now extended to a population composed of n couples (always indexed by $i = 1, ..., n$, with $n \geq 2$) and m singles (indexed by $j = 1, ..., m$ with $n \geq 2$). Let $\mathbf{y}^s$ designate a generic income vector for single individuals, rearranged in an increasing way. Its feasible set $\mathbb{Y}_m$ is

$$\mathbb{Y}_m = \left\{ \mathbf{y}^s \in \mathbb{R}_+^m \mid y_1^s \leq y_2^s ... \leq y_m^s \right\}.$$

By denoting $\mathbf{Y} = (\mathbf{y}^s, \mathbf{Y}^c,)$ a rearranged income vector of the overall population, we also define

$$\mathbb{Y} = \left\{ \mathbf{Y} \in \mathbb{R}_+^{m+n} \mid Y_1 \leq Y_2 ... \leq Y_{m+n} \right\}.$$

Given $\mathbf{Y} \in \mathbb{Y}_{n+m}$, the corresponding vector of *individual* incomes is denoted $\mathbf{y} = (\mathbf{y}^s, \mathbf{y}^c)$. To save notations, j will serve as an index for individuals as well. The set of feasible distributions of individual incomes is denoted by

$$\mathbb{Y}_{2n+m} = \left\{ \mathbf{y} \in \mathbb{R}_+^{2n+m} \quad y_1 \leq y_2 \leq .. \leq y_{2n+m} \right\}.$$

Observe that $\mathbf{y}$ contains the incomes of singles and the incomes of individuals living in a couple adjusted for public goods as defined in (2). We now strengthen the properties of g by introducing a further 'regularity' condition.

To investigate the inheritance of GL test from households to individuals is a pointless exercise whenever households have different needs, since the GL criterion becomes inappropriate for welfare comparisons. We then focus our attention on the dominance criteria proposed by Bourguignon (1989) and by Atkinson and Bourguignon (1987) (henceforth B and AB, respectively).

Assumption 1 *Let u^c and u be twice differentiable, non-decreasing and concave functions representing the utility of a couple and of an individual, respectively. We consider the following cases:*

$\bar{\mathbf{B}}$) $u^{c\prime}(z) - u'(z) \geq 0$ for all $z \geq 0$

$\overline{\mathbf{AB}}$) $u^{c\prime}(z) - u'(z) \geq 0$ and $u^{c\prime\prime}(z) - u''(z) \leq 0$, for all $z \geq 0$.

Under the assumption $\bar{\mathbf{B}}$, the difference between the utility functions of a couple and a single individual, for a given income, is a non-decreasing function. This difference becomes non-decreasing and concave under the assumption $\overline{\mathbf{AB}}$. The B and AB social dominance criteria are the following

Definition 2 *Given* $\mathbf{Y}, \mathbf{Y}' \in \mathbb{Y}_{n+m}$*,*

$\mathbf{Y}$ *dominates* $\mathbf{Y}'$ *according to the B (AB) criterion, denoted by* $\mathbf{Y} \succcurlyeq_B \mathbf{Y}'$ ($\mathbf{Y} \succcurlyeq_{AB} \mathbf{Y}'$)*, iff*

$$\sum_{i=1}^{n} u^c(Y_i) + \sum_{j=1}^{m} u(y_j^s) \geq \sum_{i=1}^{n} u^c(Y_i') + \sum_{j=1}^{m} u(y_j^{s\prime}), \tag{3}$$

for all utility functions u^c *and* u *satisfying the condition* $\bar{\boldsymbol{B}}$ ($\overline{\mathbf{AB}}$)*.*

The test associated to the AB dominance criterion is named sequential generalized Lorenz test and it is easy to implement: "take first the most deserving group, then add the next most deserving group and so on, until all groups are included, checking at each stage for GL dominance. If this obtains, one distribution can be recommended over the other" (Lambert (1993), p. 86). The Bourguignon criteria also is equivalent to an implemented algorithm, based on the Foster and Shorrocks' (1988) idea that the 'poverty gap' is always lower in the dominant distribution, whatever poverty limit that is chosen. Bourguignon criterion allows for different poverty limits among types of households, but imposing that the poverty limits are non-decreasing with needs.

It will be useful to recall the transfer criteria associated with these concepts of social dominance. Ebert (2000) clarified this topic: for an ethical observer that follows the B dominance criterion, the social welfare improves after *increments*, *progressive transfers within groups* and after *progressive transfer between groups*, that is any progressive transfers from a less deserving household to a needier one. If the normative criterion implemented by the decision maker is the AB one, then a further principle has to be added, the so called *principle of diminishing transfers between groups*. It is described by Ebert (2000) as follows: "A progressive transfer changing two given income levels within a subpopulation is relatively more desirable the needier the respective subpopulation".

3.2 Welfare analysis: from heterogeneous households to individuals

In this part of the paper we explore the possibility of the *conversion* of the welfare criteria for heterogeneous households into GL dominance among individuals. We show a positive result and a negative one. The dominance in the B sense among heterogeneous households implies GL dominance at individual level. A similar result does not hold for the AB sequential test.

Proposition 2 *If g and f_p are concave, then for all $\mathbf{Y}, \mathbf{Y}' \in \mathbb{Y}_{n+m}$*

$$\mathbf{Y} \succcurlyeq_B \mathbf{Y}' \implies \mathbf{y} \succcurlyeq_{GL} \mathbf{y}'.$$

Proof. See Appendix ■

This result is in line with the principle of progressive transfers among groups mentioned above, since a progressive transfer from a single to a couple generates a pair of progressive transfers among individuals.

A result similar to Proposition 2 cannot be guaranteed for the AB criterion, even if we omit public goods and we assume a 'very regular' sharing function, as we show in the following

Example 1 *Let us consider a first income distribution $\mathbf{Y} = (\mathbf{Y}^c, \mathbf{y}^s)$, such that $\mathbf{Y}^c = (14, 16)$ and $\mathbf{y}^s = (10, 20)$ and the income distribution $\mathbf{Y}' = (\mathbf{Y}^{c\prime}, \mathbf{y}^{s\prime})$, such that $\mathbf{Y}^{c\prime} = (10, 20)$ and $\mathbf{y}^{s\prime} = (14, 16)$. It is easy to check, using the sequential generalized Lorenz test, that $\mathbf{Y} \succcurlyeq_{AB} \mathbf{Y}'$. Assuming a perfectly egalitarian sharing function, we generate the individual income distributions $\mathbf{y} = (7, 7, 8, 8, 10, 20)$ and $\mathbf{y}' = (5, 5, 10, 10, 14, 16)$. These distributions are non-comparable by the GL criterion: even under the 'more regular' egalitarian sharing function, the AB criterion is not automatically converted into GL dominance at the level of individuals.*

The rationale behind this negative result can be explained in terms of principle of transfers. Let us consider the income distribution $\mathbf{Y}'' = (\mathbf{Y}^{c\prime\prime}, \mathbf{y}^{s\prime\prime})$, such that $\mathbf{Y}^{c\prime\prime} = (10, 20)$ and $\mathbf{y}^{s\prime\prime} = (10, 20)$. The corresponding vector of individual incomes generated by the egalitarian sharing rule is, in this case, $\mathbf{y}'' = (5, 5, 10, 10, 10, 20)$. The distribution $\mathbf{Y}$ of the example above is obtained from $\mathbf{Y}''$ by performing a progressive transfer (of 4 units of income) *within group* at the level of the more deserving group (the couples). Similarly, $\mathbf{Y}'$ is obtained from $\mathbf{Y}''$ by performing the same progressive transfer among singles, that is among individuals belonging to the less deserving group. The *principle of diminishing transfers between groups* may operate, and we consistently register $\mathbf{Y} \succcurlyeq_{AB} \mathbf{Y}'$. Nevertheless, by reasoning in terms of individual income distributions, we observe that $\mathbf{y}$ is obtained from $\mathbf{y}''$ by a couple of progressive transfers (each of 2 income units) between relatively poor individuals, whereas $\mathbf{y}'$ is obtained from $\mathbf{y}''$ by a sole transfer of 4 income units in the high part of the income distribution. The problem arises since, in the social dominance setup, we cannot motivate that the first transfer is more welfare-improving than the second one by just referring to monotonicity and concavity of individual utility functions.

4 An Empirical Test of the "Double Concavity" condition

One stake of this paper is to determine whether or not the conditions of Propositions 1 and 2 are attainable. If the household's public expenditures and the sharing rule of the dominated individual are concave with respect to household's income, then the standard method adopted by decision-makers to compare income distributions remains valid from an individual's welfare point of view. If it is not the case, one would need to adopt a new approach based on a correct representation of the intra-household income sharing process (Chiappori (1988), Browning and Chiappori (1998)).

In the following, we propose an empirical test of the double concavity condition. The test is based on the estimates of the conditional expectancies of public expenditures and of the share of income with respect to household's income. This means that some households can potentially present some non concave pattern even if the test is not rejected. Thus, it is quite standard but worth noting that a non-rejection result does no necessary mean that the theory corresponds to reality but remains a good indicator in this direction.

The empirical part takes place in three steps. The french database is first described. Then, the sharing rule is predicted, which requires some few structural assumptions that need to be described. Finally, the concavity of the predicted share going to the dominated individual and the concavity of public expenditures are tested.

4.1 Data

We use the French family expenditure survey namely *enquête budget des familles*, year 2000 for the implementation of the test. This kind of data usually presents problems due to the different purchase frequencies of goods. To prevent this problem, two data collecting methods are simultaneously used. The first one is a direct interview of the household, which aims at collecting last household's expenditures such as rent, electricity, childcare, etc., expenditures during the last 2 months (clothes, fuel, etc.) and some expenditures during the last year (service charges). Expenditures for the last two weeks are directly recorded by individuals themselves on a small book. With this method, misreporting due to remembering is minimized. On the counterpart, INSEE needs to control for seasonality in the expenditures in order to construct annual expenditures for each good category. As usual, data are collected at the household's level and we do not explicitly know who is the main beneficiary of each consumption within the household. The only category that can, to a certain extent, be assigned[6], is clothes and shoes expenditures. Apart from expenditures, net incomes, savings and socio-demographic characteristics are also collected.

Households containing more than two adults without children were excluded from the analysis. Given the identification assumptions needed below, the absence of children in the sample seems necessary. Indeed, they very probably play a role on the bargaining process (through public expenditures, their own bargaining power or parents's altruistic

[6] We use the term "assignable" to designate a private good consumption observed on an individual basis. Sometimes, clothes and shoes' expenditures cannot be assigned in the data in which case the consumption is aggregated into household's aggregate private consumption. Unassignable clothes consumption represents on average 1.5% of household's expenditures.

preferences) and this has a consequence on the structure of consumption of couples. Moreover, if we can hope that the assumption of identity of preferences for clothes between individuals living in a couple and individuals living single holds. It could rarely be the case when children are present (single mothers present a very different consumption pattern than single women and than women in a couple).

The second selection rule consists in withdrawing from the sample households containing one member aged more than 60 years old. In fact, this selection does not appear to have a big impact on the final concavity test but it has the advantage to create homogenous samples of singles. Without this selection, single living women would have been more frequently observed than single-living men and we could fear that their preferences for clothes would be slightly different than that of women observed living in a couple.

Finally, as usual, we withdraw from the sample individuals who do not consume a positive amount of assignable clothes[7]. This selection rule implies that households with a higher taste for clothes or with a higher income are selected. The neutrality of this selection rule on the analysis is assumed even if it could potentially be related to the household's decision-making process. As shown in Table 1, individual characteristics slightly differ when selecting the sub-sample. In particular, the educational level tend to be higher in the selected sample and this could potentially be related to the balance of the bargaining power within families and influences the prediction of the sharing rule (see section 3.2) towards higher equality between spouses.

After these selections, we can notice that the sample size is quite small for a non parametric analysis (461 observations for the smallest sample) and it would not have been reasonable to split samples by individual characteristics in order to homogenize further the population of singles and couples with respect to their clothes consumption taste[8]. An immediate solution one can think about in the presence of a relatively

[7]In the following, "clothes expenditures" will designate both clothes and shoes expenditures.

[8]One possibility would have been to take into account the fact that people who live in big cities tend to have a higher consumption of clothes than people who live in smaller towns, this would have been

small sample is to implement a semi-parametric analysis specifying a specific parametric shape of the effect of explanatory variables on clothes consumption but keeping the effect of income unspecified. However in this case, the theoretical results do not hold any more and the empirical approach would not have been consistent with the theoretical part.

Clothes consumption represents a relatively small share of household's expenditures (around 5%). As the share is low, results could be sensitive to the presence of small clothes consumption measurement errors. Nevertheless, this approach remains the only implementable way to predict a sharing rule (see Browning et al. (1994), Browning et al.(2003)).

$[INSERT\ TABLE2]$

Table 2 shows descriptive statistics of the sub-sample. The variation of shares between singles and couples gives an indication of the degree of publicness or economies of scales implied by the good consumption. Indeed, expenditures related to the accommodation represent a much lower share of the budget for couples than for singles, this probably reveals the public nature of the house. On the contrary, the other goods do not show a decreasing share when looking at couples instead of singles. This is the case for clothes expenditures which we can suspect to be mainly private as the share does not change much between singles and couples (forgetting about heterogeneity in preferences and income). This exploratory analysis make us choose a quite restrictive definition of public expenditures which are defined as the expenditures related to the accommodation (rent, heating, energy for the house).

As usual, the income variables of the precedent theoretical part (y or Y) are replaced with (individual's or household's) expenditure for the empirical analysis. Total household's expenditures could be endogenous because of the simultaneity of the sav-

the most important explanatory variable for clothes consumption. Preliminary exploratary analysis on a subsample of individuals who do not live in big cities do not show a change in the nature of the result.

ing and consumption decisions or because good consumptions are related to the same unobserved variables as household's total expenditures. For these reasons, endogeneity of total expenditures is controlled by using the total household gross income as an exclusion restriction.

4.2 Prediction of the sharing rule

4.2.1 Identification assumptions

The allocation of private expenditures within the family is usually unobserved in the data. In order to test for the concavity of the share going to the *dominated* individual (p_i) in the family, we need to predict this share. For this purpose, and to remain consistent with the theoretical part, we assume that the unknown shape of the sharing rule is the same for every household. After having considered that clothes consumption is an assignable good, *i.e.* a good which is privately consumed by individuals within the family, we adopt an identification method close to Browning, Chiappori and Lewbel (2003).

As one could notice, this set of assumptions is quite strong. However, it corresponds to what is currently done in the literature. If we can deplore the eventual presence of externalities of clothes consumption (one could care about his or her spouse appearance), it is also probable that individual preferences for clothes, with respect to other goods, change when getting married or when getting divorced. Another possibility is that the marriage market selects individuals who present different preferences for clothes and thus can be directly (or indirectly through covariates) related to the intra-household sharing rule. In all these cases, the prediction of the sharing rule can be biased. The weakest set of identifying assumptions in the labor supply context can be found in Laisney (2002), who considers the possible interaction between leisure time of spouses and in Couprie (2002), who takes into account the possible selection bias by using panel data on marital status changes. However, these work cannot be useful here as we are primary interested in the analysis of consumption and as there do not exist any panel data on consumption.

4.2.2 Implementation

We denote the household by $i = 1, ..., n$ and the individual within the household by $j = f, m$, respectively for females and males. In this general approach, the 'dominant' and the 'dominated' individual in the family can be either the female or the male. Total household expenditures Y are split between public expenditures, G, and private expenditures, $\widetilde{Y}$:

$$Y_i = \widetilde{Y}_i + G_i. \tag{4}$$

Private expenditures are then split into assignable private consumptions of clothes c_i^f and c_i^m and a non assignable Hicksian-aggregated private consumption good. In the absence of price variations, we can infer the intra-household share of the good by observing both the share of the assignable consumption good expenditures within couples and the Engel curve of clothes expenditures of single individuals. A necessary condition is that individuals must allocate their expenditures for clothes and the Hicksian aggregated good the same way whether there are living alone or in a couple.

This assumption requires a lot. In particular, it requires that the mariage market does not select individuals who express different preferences for clothes (lower or higher) with respect to the other goods, or who have different and unobserved individual characteristics which also explains individual's preferences for clothes. It also requires that *preferences for clothes* do not change when getting married. On the other hand, this approach does not require *identical preferences* across marital status.

Individual expenditures y_i^j, net of accomodation-related expenditures, are directly observed when an individual lives single. Let's assume that the Engel curve for clothes consumption has the following non parametric shape, g^f for females and g^m for males:

$$c^j = g^j \left(y^j\right) + e^j \quad \text{with } E(e^j/w = 0) \text{ and } j = f, m. \tag{5}$$

Total private expenditures y could be endogenous and there exists some exogenous instrumental variable w,which his basically the gross household income. The e_i are independently and identically distributed between households and have the following properties: $E(e^j/w) = 0$ and $Var(e^j) = \sigma_j^2 I$.

In the exogeneity case, $E(e^j/y^j) = 0$, a consistent non parametric kernel estimator $\widehat{g^j}$ of the Engel curve can be obtained on the sub-sample of single individuals. In the following, we implement a Nadaraya-Watson conditional moment estimator:

$$\widehat{g^j}(y)_{exo} = \frac{\sum_{i=1}^{n_j} K\left(\dfrac{y_i - y}{h}\right) c_i}{\sum_{i=1}^{n_j} K\left(\dfrac{y_i - y}{h}\right)}, \tag{6}$$

with K a well-behaved quartic kernel function, h the bandwidth, n_f the sample size of single-living women and n_m the sample size of single-living men. The asymptotic properties of this estimator are surveyed in Härdle and Linton (1994). The bandwith was chosen according to Silverman's rule of thumb.

In the endogeneity case this estimator is not convergent anymore. Furthermore, the first moment condition $E(e^j/w) = 0$ does not give enough information to non-parametricaly identify g^j so that we need to introduce somehow some parametric assumptions. By precising the shape of the correlation between the instrument and the dependent variable, we can simplify this problem. As in Blundell, Browning and Crawford (1997), we assume that:

$$c = g^j(y) + \rho\nu + u \text{ with } E(u/y = 0), \tag{7}$$

where ν is the error term resulting from the instrumental equation (8), orthogonal with respect to the instrumental variable(s) $E(\nu/w) = 0$. The instrumental equation is:

$$y = w\delta + \nu. \tag{8}$$

The kernel consistent estimator is then obtained as a simple form of the kernel estimator in the exogenous case:

$$\widehat{g^j}(y)_{endo} = \widehat{g^j}(y)_{exo}.\widehat{\rho}, \tag{9}$$

where $\widehat{\rho}$ is obtained as the effect of the residual of the instrumental equation on clothes consumption.

To globally inverse the Engel curve, the monotonicity is required. We ensure the monotonicity of $\widehat{g}$ by imposing a shape-restriction on the Kernel regression estimator (see Matzkin (1994) and Mukarjee and Stern (1994)). Basically, the monotonicity-constrained estimator is an arithmetic average of an upward and a backward estimator, its computation has the advantage to be quite fast:

$$\widehat{g}^j(y) = \frac{\widehat{g}_U(y) + \widehat{g}_L(y)}{2}, \tag{10}$$

with:

$$\begin{cases} \widehat{g}_U(y) = \min\limits_{y' \geq y} \widehat{g}^j(y') \\ \widehat{g}_L(y) = \max\limits_{y' \leq y} \widehat{g}^j(y') \end{cases}. \tag{11}$$

The asymptotic distribution of this shape-restricted estimator is not yet established so that we cannot implement a global test of the constrained estimator against the unconstrained one. Nevertheless, we can implement a local test of the monotonicity assumption by first estimating and testing the sign of the derivative at each point of the grid[9]. The local linear derivative estimator has the following expression[10]:

$$\widehat{g}^j(y) = \frac{\sum_{i=1}^{n_j} K^{(1)}\left(\frac{y_i - y}{h}\right) c_i}{\sum_{i=1}^{n_j} K^{(1)}\left(\frac{y_i - y}{h}\right)}, \tag{12}$$

where $K^{(1)}$ represents the first derivative of the kernel with respect to y. Under i.i.d. assumptions, this estimator follows asymptotically, at the convergence rate of n, a normal distribution and its 5% confidence band can be calculated as follows:

$$\widehat{g}^j(y) \pm 1,96 \sqrt{\frac{1}{h}\left(\widehat{\sigma}^2 \frac{\int \left(K^{(1)}(\psi)\right)^2 d\psi}{\Sigma_{i=1}^{n_j} K^{(1)}\left(\frac{y_i - y}{h}\right)}\right)}, \tag{13}$$

[9]A global monotonicity test exists but is not informative. Indeed, it is precisely required that the monotonicity at each point is satisfied.

[10]Pagan and Ullah (1999) survey all kind of local and global derivatives estimators.

with

$$\widehat{\sigma}^2 = \frac{\Sigma_{i=1}^{n_j} \left(c_i - \widehat{g}^j\left(y\right)\right)^2 K\left(\dfrac{y_i - y}{h}\right)}{\Sigma_{i=1}^{n_j} K\left(\dfrac{y_i - y}{h}\right)}. \tag{14}$$

Let's denote $\widehat{g_j^{-1}}(c_i)$ the inverse of the Engel curve giving total predicted private expenditures (at the conditional expectancy) as a function of clothes consumption. Under the common support assumption between clothes expenditures of women (resp. men) in a couple and single women (resp. men), private expenditures of individuals living in a family can be predicted by inversing the Engel curve of clothes consumption predicted for single individuals on individuals living in a couple. The predicted share of private expenditures of individuals living within a family is then deduced as:

$$\begin{cases} \widehat{y^f} = \widehat{g_f^{-1}}(c^f) \\ \widehat{y^m} = \widehat{g_m^{-1}}(c_m) \end{cases}. \tag{15}$$

By this mean, one share is deduced from the inversion of female's clothe expenditures (Case A) and the other from the inversion of male's clothes expenditures (Case B). When both spouses clothes consumption belong to the common support, the individual share is overidentified (Case C). In this last case the estimator naturally gains in precision if we average other both estimates.

4.2.3 Sharing rule prediction results

[*INSERT FIGURES* 1]

Figures 1.a and b show the monotonic non parametric estimator of single individuals' Engel curves of clothes consumption. Estimations were implemented on a sub-sample excluding the 5% extreme total expenditures in order to prevent from measurement errors at the top and the bottom of the income distribution. Figures representing the unconstrained estimator are given in Appendix A.1 and A3. More interesting is the local estimate of the derivative of the unconstrained estimator (Figures A2 and A4). The 5% confidence band can be used to test if the monotonicity statisticaly holds at each point of the grid. It appears that except around 20000 euros per year for males, the positivity of the derivative of the Engel curve cannot be statistically rejected (the upper band is always above the 0 line). This seems satisfactory enough to implement the constrained estimate.

The endogeneity of household private expenditures was rejected for all the regressions. It is possible that this comes from the restrictive shape imposed on the endogeneity bias (see equ. 7). In any case, it remains that controlling for endogeneity in a non-parametric way is not trivial and generates some difficulties from an econometric perpective.[11]

Because of the common support assumption required by the non parametric prediction of the sharing rule, the identification can be implemented on three different sub-samples. The sub-sample A (resp. B) contains observations where female's (resp. male's) Engel curve is inversed. Women (resp. men) in a couple's clothes consumption must belong to the support of the predicted Engel curve of single women (resp. men). The sub-sample C combines female's and male's predictions and is naturally smaller than sub-samples A and B. On the counterpart, the prediction, given the sample size, should be more precise.

[$INSERT\ TABLE$ 3]

In order to have an idea of the potential selection bias related to the common support assumption, Table 3 describes the characteristics of sub-samples A, B and C. As we can notice in this table, average values of covariates tend to change as well as the predictions of the sharing rule going to women living in a couple. Comparing Table 3 with statistics from couples in Table 2, it is clear that the common support assumption generally tends to select couples with lower total expenditures. Sub-sample C

[11] Even if the approach undertaken here does not seem to satisfactory control for the endogenity of total expenditures, there is not, at that time, clear evidence in the literature about the importance of endogeneity of total expenditures in consumption systems when the relationship between good expenditures and total expenditures is non parametric. These important aspects are left for further research.

under-estimates from about 15% the average of total household expenditures compared to the original sample. From clothes expenditures point of view, variations between sub-samples remain quite high. Sub-sample A contains couples which over-consume clothes, whereas sub-sample C contains couples which under-consume clothes. Indeed, sub-sample C is a compromise between sub-sample A and B as female's clothes are slightly over-consumed (around 6%) and male's clothes under-consumed (around 11%) compared to the original sample. These differences could have an impact on the predicted sharing rule by sub-sample C which could probably be slightly over-estimated by this method. Even though this probable over-estimation, due to some selection bias, on the average share could be thought as harmful, what really matters for the following concavity test is not the level of the share but the relationship between total household expenditures and the share going to the *dominated* individual. There is no a priori reason why this relationship would be biased when selecting couples characterized by differing preferences for clothes.

Looking at the sharing rule predictions expressed in ratios, it appears that the three samples lead to a female share going from 52 to 56% of household's expenditures on average. It is clear from Figure 2 that sub-sample C gives a more satisfactory picture than A and B on what could be the intra-household sharing rule (the ratio rarely goes beyond 0 or above 1). This sub-sample will be used from now on to implement the concavity test on the *sharing rule*.

4.3 Testing the double concavity

A double concavity test is implemented on both the *public sharing function* and the *private sharing function*. The first step does not require any particular assumption whereas the second does in order to identify the degree of intra-household inequality (see above). The non parametric concavity test implemented here has been recently developped by Abrevaya and Jiang (2004) in the multivariate case. One particularity of this test is that it does not rely on a comparison between a constrained and an unconstrained model. The asymptotic properties of the test are directly derived from the simplex statistic and this only requires a minimalist set of assumptions. Apart from the i.i.d. assumption, this test requires the symmetry of the distribution of the residual. In the univariate case, the mechanism of the test is simple as it consists in checking for the concavity of the function for each possible 3-tuple of the sample. The potential combinations are huge but the statistics of the test is directly derived from the mathematical definition of the concavity without having to specify any specific relation between the sharing function and total household expenditures.

In our case, the test statistics takes the following formulation:

$$U_n = \left(C_n^3\right)^{-1} \left[\# \text{ of convex 3-tuples - } \# \text{ of concave 3-tuples}\right], \tag{16}$$

where n is the sample size and C_n^3 represents the number of 3-tuples in the sample. This statistics asymptotically follows a normal distribution with variance ζ that can be approximated by the following estimator:

$$\widehat{\zeta} = \frac{9}{n} \Sigma_{t=1}^{n} \left(U_t - U_n\right)^2. \tag{17}$$

$[INSERT\ FIGURE\ 3]$

If we first take a look at the dependence of public expenditures with respect to household's total expenditures, the concave relationship seems to hold for households with expenditures lower than 40000 euros per year. It appears indeed that the test does not reject the concavity of the function for all the individuals and a fortiori for households with lower expenditures. 84.7% of 3-tuples respect the (strict) concavity condition. The nul hypothesis is that $U_n \leq 0$, the test statistic is $S = -0.43$ whereas the p-value appears around 0.66 which clearly does not reject the concavity against the convexity.

$[INSERT\ FIGURE\ 4]$

By defining the *dominated* individual as the individual who benefits from less than 50% of household's private expenditures, a relation that looks somehow linear is observed between the share of the *dominated* and household's private expenditures. Refining this observation by a formal test indicates that 41.8% of 3-tuples show a concave shape against 58.2% a convex shape. The statistic which is $U_n = 0.17$ does not reject concavity (when the concavity is the null hypothesis,the p-value is 0.43), moreover it does not reject linearity (when the linearity is the null hypothesis, the p-value is around 0.86). Given the non-parametric test presented above, the double concavity test is not rejected on our data. Moreover, the sharing rule appears linear with income which implies that the household income level does not have on average any effect on the level of intra-household inequality (at least couples without children selected for the analysis).

5 Concluding remarks

Our aim was to look if welfare comparisons made at the household could be translated on an individual basis. Our temporary conclusion is that, at this stage, we cannot reject the assumption that the Generalized Lorenz curve which is a helpful graphical device, retains its normative meaning, in terms of inequality and welfare at the individual level as it has at the household's one, at least in the French example in 2000. In other words, it may be sufficient to check the dominance at the household level to conclude at a decrease of inequality among individuals.

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Appendix

Proof of Proposition 1

Suppose that g and f_p are non-decreasing and concave and consider $\mathbf{Y}, \mathbf{Y}' \in \mathbb{Y}_n$ such that $\mathbf{Y} \succcurlyeq_{GL} \mathbf{Y}'$. We prove that

$$\sum_{i=1}^{n} \left[u(y_{ip}) + u(y_{ir})\right] \geq \sum_{i=1}^{n} \left[u(y'_{ip}) + u(y'_{ir})\right],$$

for all u non-decreasing and concave, which is equivalent to $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$. For a given individual utility function u, we denote w the sum of individual utilities in the household i, that is $w(Y_i) = u(y_{ip}) + u(y_{ir})$. By omitting the index i and using the fact that all functions are twice differentiable, we prove as an intermediate result that $w'(Y) \geq 0$ and $w''(Y) \leq 0$, $\forall Y \geq 0$.[12] We get

[12] See Peluso and Trannoy (2004) for a different proof in the non differentiable case.

$w'(Y) = u'(y_p)[g'(Y) + f_p'(\tilde{Y})(1 - g'(Y))] + u'(y_r)[g'(Y) + f_r'(\tilde{Y})(1 - g'(Y))]$. Since $0 \leq g'(Y) \leq 1$, it is easy to see that this expression is non-negative.

By posing $A = g'(Y) + f_p'(\tilde{Y})(1 - g'(Y))$ and $B = g'(Y) + f_r'(\tilde{Y})(1 - g'(Y))$, we get $w''(Y) = u''(y_p)A^2 + u''(y_r)B^2 + u'(y_p)A' + u'(y_r)B'$.

The first two terms are non-positive by assumption. After some manipulations, we get $u'(y_p)A' + u'(y_r)B' = f_p''(\tilde{Y})[1 - g'(Y)]^2[u'(y_p) - u'(y_r)] + g''(y)[u'(y_p)f_r'(\tilde{Y}) + u'(y_r)f_p'(\tilde{Y})]$. This expression also is non-positive and we may conclude the that $w''(Y) \leq 0$. Finally, from $\mathbf{Y} \succcurlyeq_{GL} \mathbf{Y}'$, we deduce $\sum_{i=1}^n w(Y_i) \geq \sum_{i=1}^n w(Y_i')$ and therefore $\sum_{i=1}^n \left[u(y_{ip}) + u(y_{ir})\right] \geq \sum_{i=1}^n \left[u(y_{ip}') + u(y_{ir}')\right]$. The reasoning is valid for all u non-decreasing and concave.

Proof of Proposition 2

We show that

$[f_p \text{ and } g \text{ concave}] \implies \left[\text{not } \mathbf{y} \succcurlyeq_{GL} \mathbf{y} \implies \text{not } \mathbf{Y} \succcurlyeq_{B} \mathbf{Y}'\right]$.

Suppose that $\mathbf{y} \succcurlyeq_{GL} \mathbf{y}'$ does not hold. Then there exists a non-decreasing and concave utility function $\tilde{u}$ such that:

$$\sum_{i=1}^n \tilde{u}(f_p(Y_i^c)) + \sum_{i=1}^n \tilde{u}(f_r(Y_i^c)) + \sum_{j=1}^m \tilde{u}(y_j^s) < \sum_{i=1}^n \tilde{u}(f_p(Y_i^{c\prime})) + \sum_{i=1}^n \tilde{u}(f_r(Y_i^{c\prime})) + \sum_{j=1}^m \tilde{u}(y_j^{s\prime}) \tag{18}$$

which turns to be equivalent to: $\sum_{i=1}^n \tilde{u}^c(Y_i^c) + \sum_{j=1}^m \tilde{u}(y_j^s) < \sum_{i=1}^n \tilde{u}^c(Y_i^{c\prime}) + \sum_{j=1}^m \tilde{u}(x_j^s)$, where $\tilde{u}^c(Y_i^c) = \tilde{u}(f_p(Y_i^c)) + \tilde{u}(f_r(Y_i^c))$. If f_p and g are concave, by using differentiability and reasoning as in the proof of Proposition 1 it is possible to show that $\tilde{u}^c$ is non-decreasing and concave. Moreover, since $\tilde{u}^{c\prime}(Y) = \tilde{u}'(y_p)[g'(Y) + f_p'(\tilde{Y})(1 - g'(Y))] + \tilde{u}'(y_r)[g'(Y) + f_r'(\tilde{Y})(1 - g'(Y))]$, it is a weigthed mean of $\tilde{u}'(y_p)$ and $\tilde{u}'(y_r)$ and it is easy to see that $\tilde{u}^{c\prime}(Y) \geq \tilde{u}'(Y)\ \forall Y \geq 0$. Then $\tilde{u}^c$ may be interpreted as a non-decreasing and concave household utility function satisfying Assumption $\bar{\mathbf{B}}$. By comparing (3) and (18), we conclude that $\mathbf{Y} \succcurlyeq_B \mathbf{Y}'$ is negated.

Table 1: Sub-sample selection

	All families (couples or singles)	**Without children, excluding elderly**	**Consuming assignable Clothes**
Number of observations	9962	2750	1794
Number of single men	1114	688	461
Number of single women	2281	711	569
Number of couples	6567	1351	764
Household's share of clothes expenditures			
- women	0.0197 (0.0331)	0.0219 (0.0359)	0.0302 (0.0401)
- men	0.0155 (0.0299)	0.0181 (0.0288)	0.0259 (0.0376)
- children	0.0094 (0.0250)	0.0002 (0.0031)	0.0003 (0.0038)
- unassignable*	0.0122 (0.0316)	0.0127 (0.0362)	0.0135 (0.0357)
Household's total expenditures (in euros/year)	24769.1 (16632.8)	22041.5 (14703.0)	22446.1 (14696.0)
Household before tax income (in euros/year)	28717.8 (21263.1)	25349,5 (18859.8)	25316.2 (19601.5)
Age of household's head	50.98 (16.74)	41.45 (12.55)	39.76 (12.51)
Education level (1 to 5)	2.88 (1.39)	3.23 (1.44)	3.39 (1.45)
Household has a child	0.31 (0.46)		
Number of children if children	1.74 (0.81)		

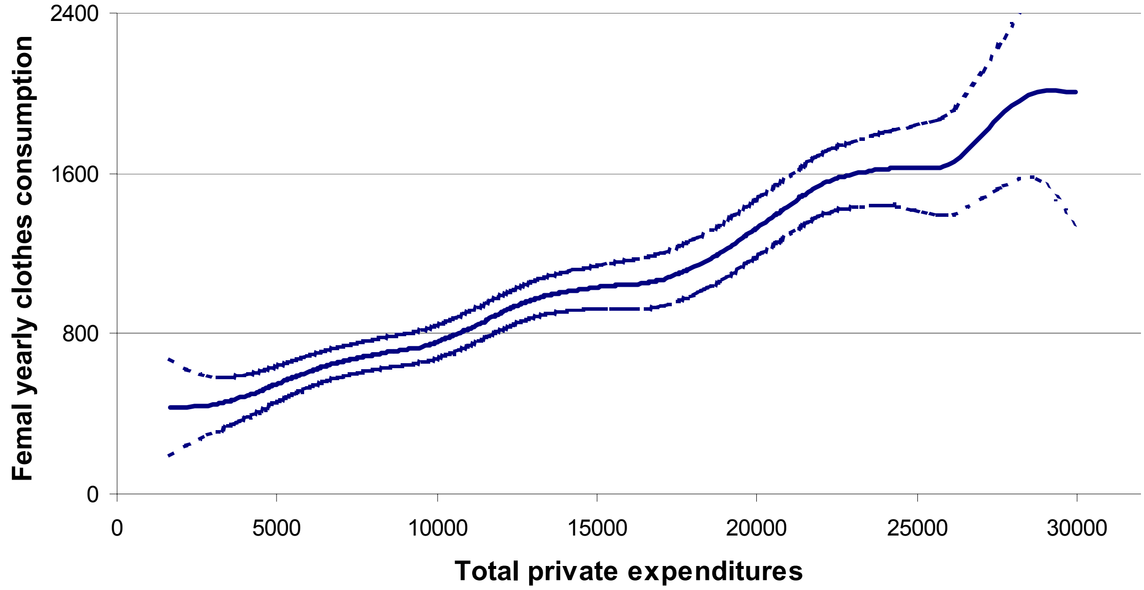
** In the following, this category will be included in other private expenditures.*

Table 2: Descriptive statistics of the sub-sample

	Single men	Single women	Couples
A/ Shares of household annual expenditures (in %)			
Accommodation and energy (**Public**)	24.51	26.61	16.61
Furnitures for the house (including domestic services)	3.11	3.50	4.70
Small furnitures for the house	5.10	4.82	5.81
Car buying	3.22	3.28	6.67
Gasoline and car-related expenditures	12.38	9.18	11.22
Leisure (hotels, restaurants, …)	13.89	12.52	11.35
Health and body	3.81	8.77	8.06
Food at home	11.03	13.18	14.69
Vices	4.41	2.59	3.18
Clothes	5.47	5.82	5.49
Other expenditures (bank, insurance, food at work, transfers…)	13.07	9.73	12.22
B/ Income and expenditures (in euros/year)			
Before tax income	18647.5	16658.5	35788.0
	(13124.9)	(10313.7)	(22962.6)
Total Expenditures	16373.3	16016.8	30898.8
	(8809.4)	(8217.4)	(16906.1)
Women' clothes expenditures	2.7050	892.23	815.09
	(39.7841)	(966.31)	(812.49)
Men' clothes expenditures	892.83	7.963	821.68
	(1181.86)	(96.053)	(941.80)
C/ Covariates			
Age of household's head	37.50	38.19	42.30
	(10.84)	(12.99)	(12.64)
Education level (1 to 5)	3.29	3.59	3.29
	(1.55)	(1.47)	(1.36)
Number of observations	**461**	**569**	**764**

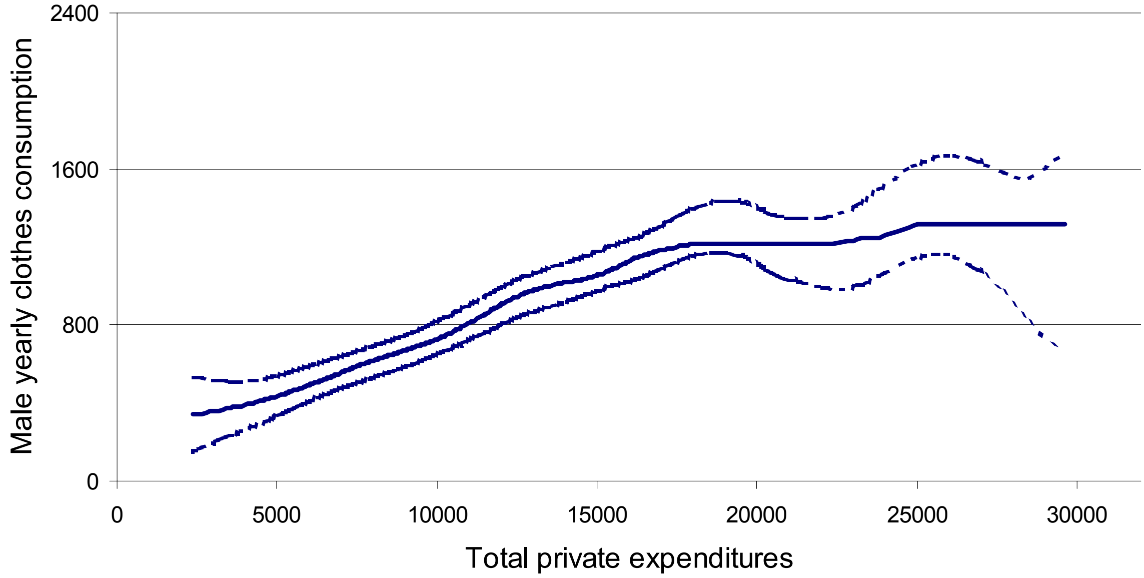
Figure 1: Clothes consumption Engel Curves estimates for single individuals

Figure 1.a: Single women



**The 5% confidence band corresponds to the unconstrained model*

Figure 1.b: Single men



**The 5% confidence band corresponds to the unconstrained model*

Table 3: Sharing rule Identification

Identification based on		(A)	(B)	(C)
Female's private expenditures				
	Mean	11993.4	15773.5	13473.6
	Standard Error	(7406.4)	(11959.0)	(7337.7)
Female's ratio of household's private expenditures				
	Min	0.0664	-0.7673	-0.1892
	Max	2.1062	0.9419	1.0882
	Mean	0.5327	0.5599	0.5278
	Standard Error	(0.3931)	(0.2965)	(0.1977)
Other household's covariates				
	Before tax income	37438.1 (21868.7)	35846.9 (21410.9)	30141.5 (12376.2)
	Total expenditures	30540.1 (12299.7)	29824.0 (12202.0)	25686.9 (11671.1)
	Female's clothes consumption	912.07 (409.66)	752.71 (660.85)	866.49 (388.42)
	Male's clothes consumption	869.38 (842.63)	697.69 (271.01)	724.78 (273.52)
Number of observations		**404**	**382**	**241**

(A) Female's Engel curve only
(B) Male's Engel curve only
(C) Both Engel curves

Figure 2: Distribution of the predicted female sharing rule

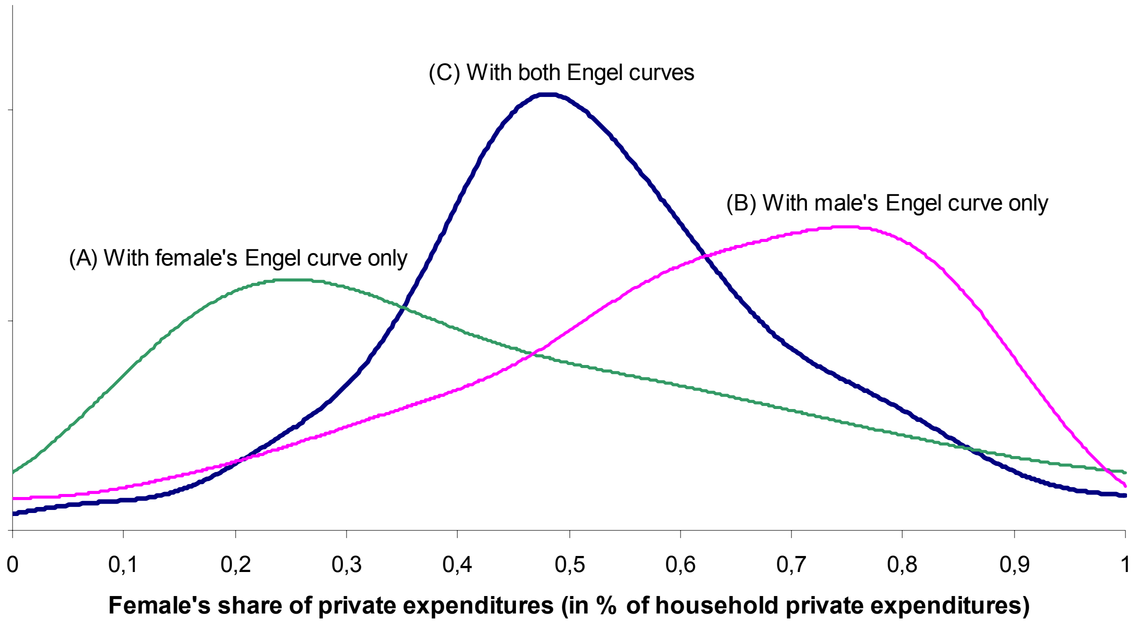
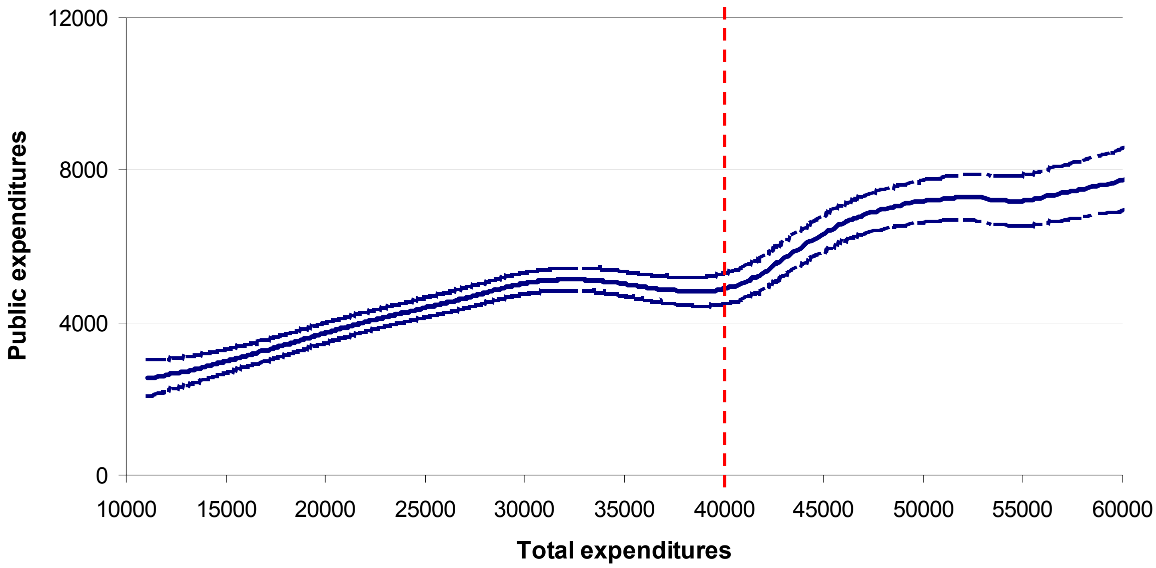
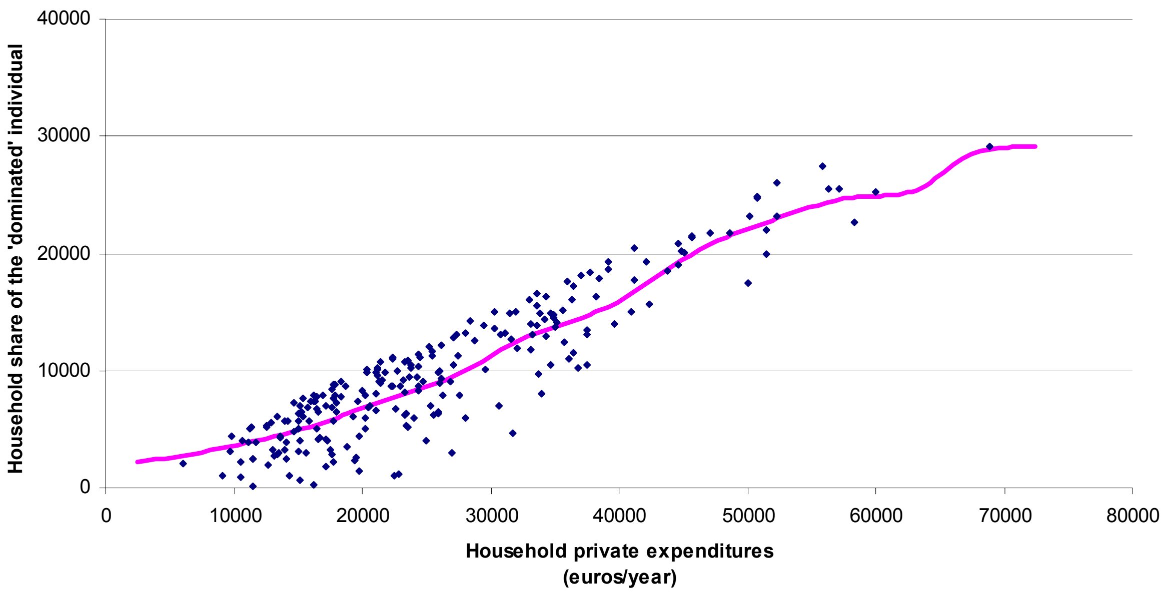


Figure 3: Kernel regression of public expenditures on household's total expenditures



* *From sub-sample(C)*

Figure 4: Share of the *dominated* on household's private expenditures



* *From sub-sample(C)*

APPENDIX

Figure A.1 Female's Engel curves of clothes consumption (unconstrained estimation)

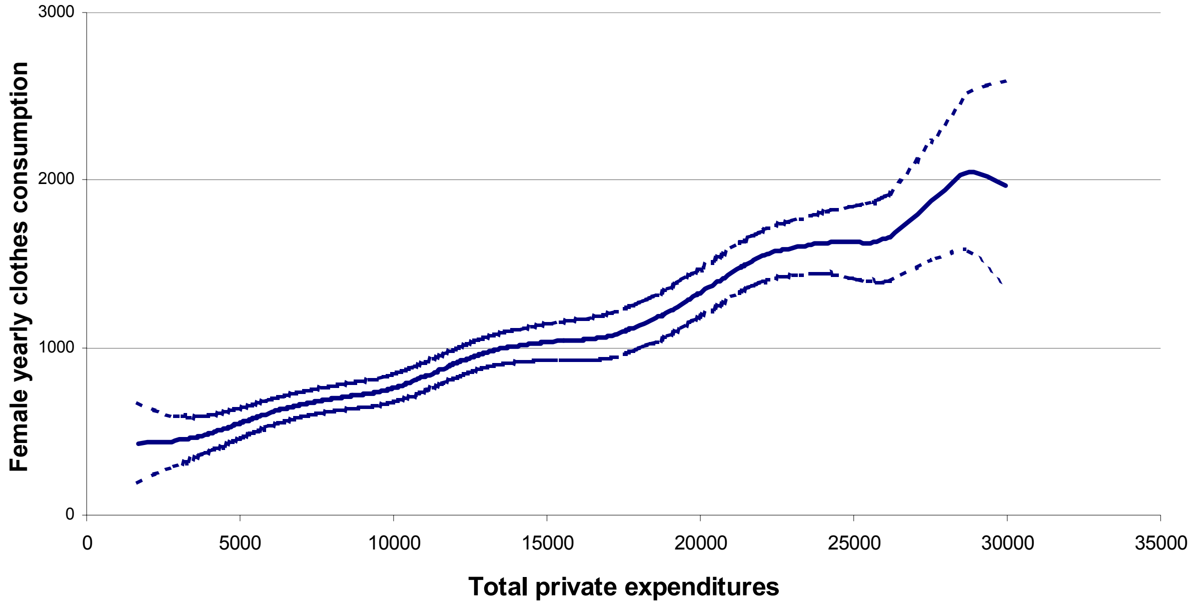


Figure A.2 Unconstrained estimation of female Engel curve derivative

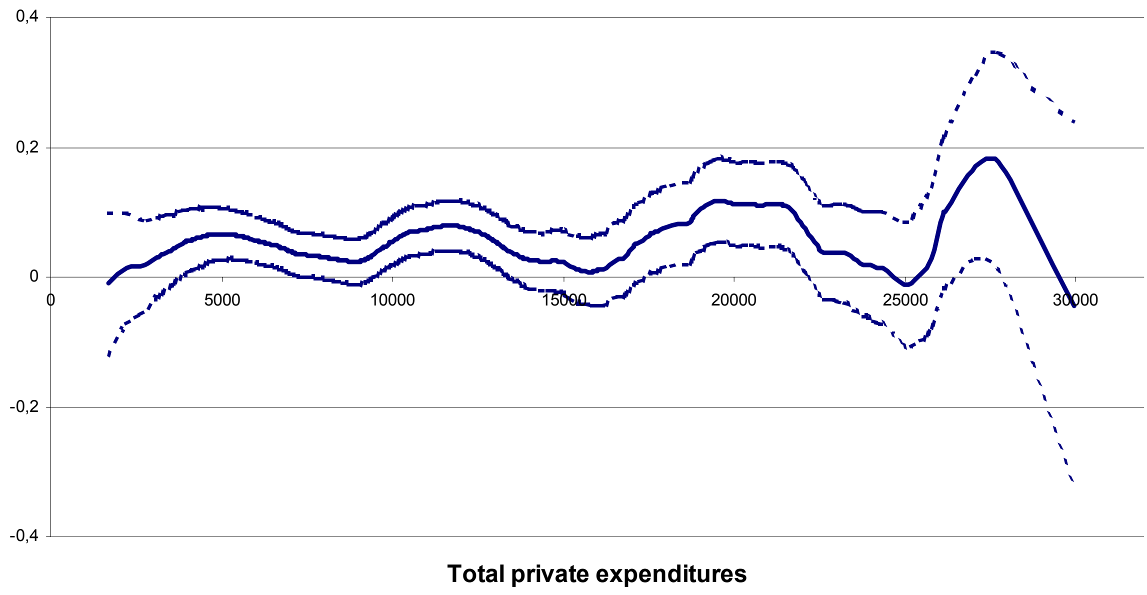


Figure A.3 Male's Engel curves of clothes consumption (unconstrained estimation)

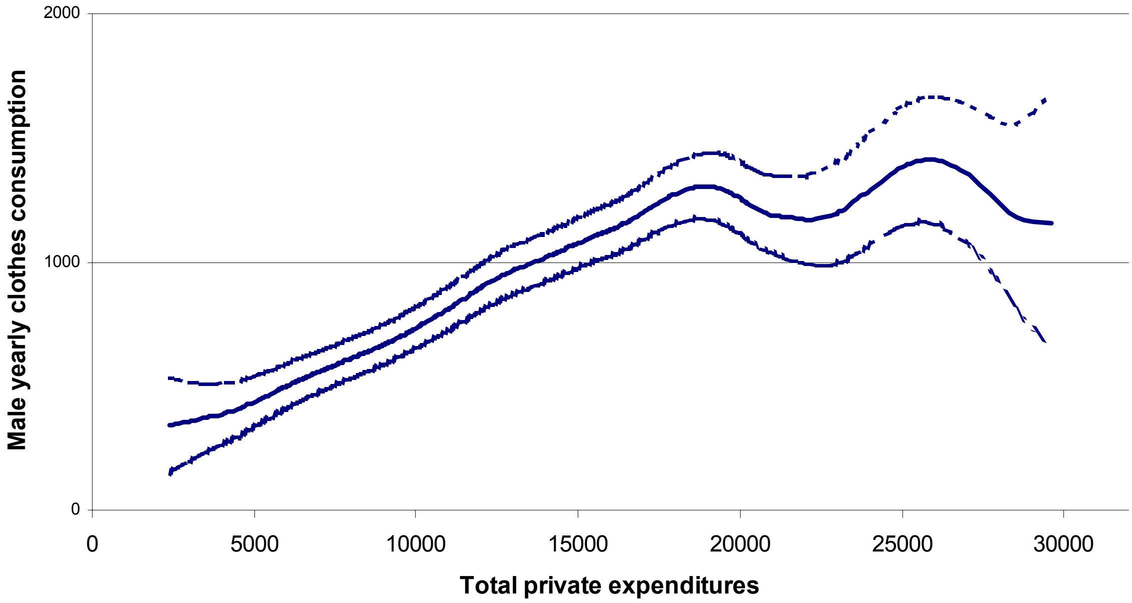


Figure A.4 Unconstrained estimation of male Engel curve derivative